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Toward a Quantitative Theory of Intellectual Discovery (Especially in Physics)

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Abstract—By the study of time intervals in a subjective yet consistently chosen temporally ordered list of the critical ideas which comprise Physics, a quantitative theory of the growth of these ideas is inferred which takes the entirely plausible form that the rate of growth of ideas is proportional to the totality of known ideas multiplied by the totality of people in the world. There is some slight titillating indication in the data that the rate of fundamental discovery in Physics has been decreasing abnormally over the past 50 years.

Introduction

Humankind have never ceased to marvel at the flow of their ideas, and to wonder about the process. Of late the focus has been on the seemingly accelerated pace of acquisition of knowledge, and many reasons have been advanced, ranging from the very perceptive observation of Henry Adams (1918) "ideas beget ideas" to the fatuous suggestion that people are getting smarter. Rescher (1978) has analyzed the existing body of thought about this question in a recent book. In a series of papers, the author, (Fowler, 1982, 1983) has shown that the proper variable in terms of which human questions should be discussed is not calendar time, which is an arbitrary and hence largely meaningless idea, but rather population numbers. In this light it will be shown that a quantitative formulation of the problem of idea growth may be possible. The development of this concept has depended upon data taken from the special body of knowledge known as Physics, and its applicability may be limited to that field. Any further generalization to other or larger spheres of ideas will be left to others.

Idea Magnitudes

Knowledge consists of an enormous number of discrete and overlapping ideas. To form a complete list even in a well defined field such as Physics defies the effort of one person, and would amount to compiling the indexes of perhaps a hundred thousand books. We require such a list of ideas and the point in time at which they occurred if we are to make any progress with this problem. Ideas, like stars, are not all of equal brilliance. To deny this is to argue against the validity of Nobel Prizes. We will use this generally accepted fact to render our problem tractable. First magnitude ideas, like stars, occur in clusters of lesser ideas which, unlike stars, usually have an intimate relation to the fruition of the "great" idea. In making the list given in the Appendix, the author drew on the experience of the many writers over the past century who have selected topics for their general physics texts, as well as upon the insights in topic selection of the numerous historians of the field. A few subjective rules were employed in making the list:

1. A major idea is one without which no further progress in all or one of the major subdivisions of the understanding of Nature could be made.
2. The list should be sufficient as a basis for a complete beginning course or an encyclopedia article.
3. In dealing with a cluster of discoveries like the mesons, the first discovery/idea was included because it was first. The second was also included because it showed multiplicity. But no more ideas from that family were counted until a novel or unifying concept was involved.
4. Each principle included was chained backward as well as forward in order to answer the question of what crucial concepts must have been known before it could be perceived.

A similar list of major ideas that appeared over a shorter time span was prepared by Auerbach (1910). Auerbach's list, continuing the stellar analogy, might be called a list to the second magnitude (500 items in 150 years). After augmenting Auerbach's detailed list, Rainoff (1929) proposed that creativity had been subject to wavelike fluctuations over this period. The perspective of this present article is to consider a wider sweep of history. While it is not essential that the list should contain all of the topics requisite to its full elucidation, it is important that they be of equal magnitude. It should be remarked that Rainoff's hypothesis is not borne out by the analysis presented here.

In assessing the merit of the conclusions reached in this paper it is important for the reader to know the origin of the idea list on which they are based. Had it been derived purely for the purpose of this analysis itself, it would be easy to discard the result on grounds of special pleading in selection of the items. Such however was not the case. The initial list, comprising 90% of the final items, was prepared by the author in 1966 in response to a request from the editors of World Book Encyclopedia to assist them in analyzing the completeness of their coverage of the discipline of Physics. Only some time afterwards did the question proposed in the introduction occur to the author. Subsequently, certain additions to the original list were made for two reasons. First, when the problem was attacked seriously in 1979, a few essential ancient ideas which are not specifically "physical" in nature and a very few overlooked modern ideas were added (marked with a *). Second, in 1982 a conscious attempt (marked with a #) was made with the cautiously solicited advice of others to overcome the apparent deficiency in the numbers of ideas since

1920. Neither set of modifications alters substantially the basic thrust of this paper, nor was the contemporary deficiency removed.

Relationship of Ideas to Population

Historians supply us with dates for the various discoveries, but all that is revealed by using time as an independent variable is the well known observation that ideas are coming faster and faster to mankind. It is only when we compare our list to the populations prevailing at the time of the discovery that real understanding emerges. To obtain population numbers, we turn to the many demographers who have made studies, estimates, and good guesses. I have analyzed these in the previously cited article (Fowler, 1983) where I showed that the simple empirical relation (N = global population)

$$\mathrm{d}N/\mathrm{d}t = \alpha N^2 \qquad (1)$$

has in recent times, and perhaps even for a million years, described human world population growth far better than the conventionally invoked exponential law. In all that time, it appears, the constant α has had only two values, 5.08×10^{-12} after the Renaissance, and 2.1×10^{-12} before. Using this second value for smoothing and extrapolation to ancient times, populations prevailing at any discovery time have been calculated and are given in the table in the Appendix.

I was first led toward the thought that there might be something interesting to be said about idea growth by having made a crude plot of the mean of the time intervals to the discovery next before and next after a given discovery against the population prevailing. A few samples of points generated by this rough method are given as hollow circles in Fig. 1 to show that even it foreshadows the final result. Curvefitting the complete set of this crude data on the period between ideas yields the power law $T = 2.5 \times 10^{18}/N^2$ which does not fit badly over three orders of magnitude in N. T is directly related to the reciprocal rate of growth of **I**, the idea total; that is to say, to $\mathrm{d}I/\mathrm{d}t$, but only loosely because of the inordinately large steps used in the early years, and so the precise relation above does not stand up in the final analysis. To obtain a better result we need to apply methods of finite differences.

The raggedness of the previous plot came because it is obtained by differentiation of raw data. A better method is to smooth the data first by forming the sum of the number of ideas prior to any given population level as shown in Fig. 2. This curve too is a power law $\mathbf{I} = kN^a$, but is much more clearly defined, with two regions, in which k and a take the value pairs (.0156, .335) and (1.675×10^{-5}, .714) respectively. Since we know the temporal dependence of population, we can use the integrated curve first to determine the expected behavior of the $\mathrm{d}I/\mathrm{d}t$ curve, and later to assist us in the calculation of more accurate data points for that curve. Thus

$$\mathrm{d}I/\mathrm{d}t = kaN^{(a-1)}(\mathrm{d}N/\mathrm{d}t) = ka\alpha N^{(a+1)}. \qquad (2)$$

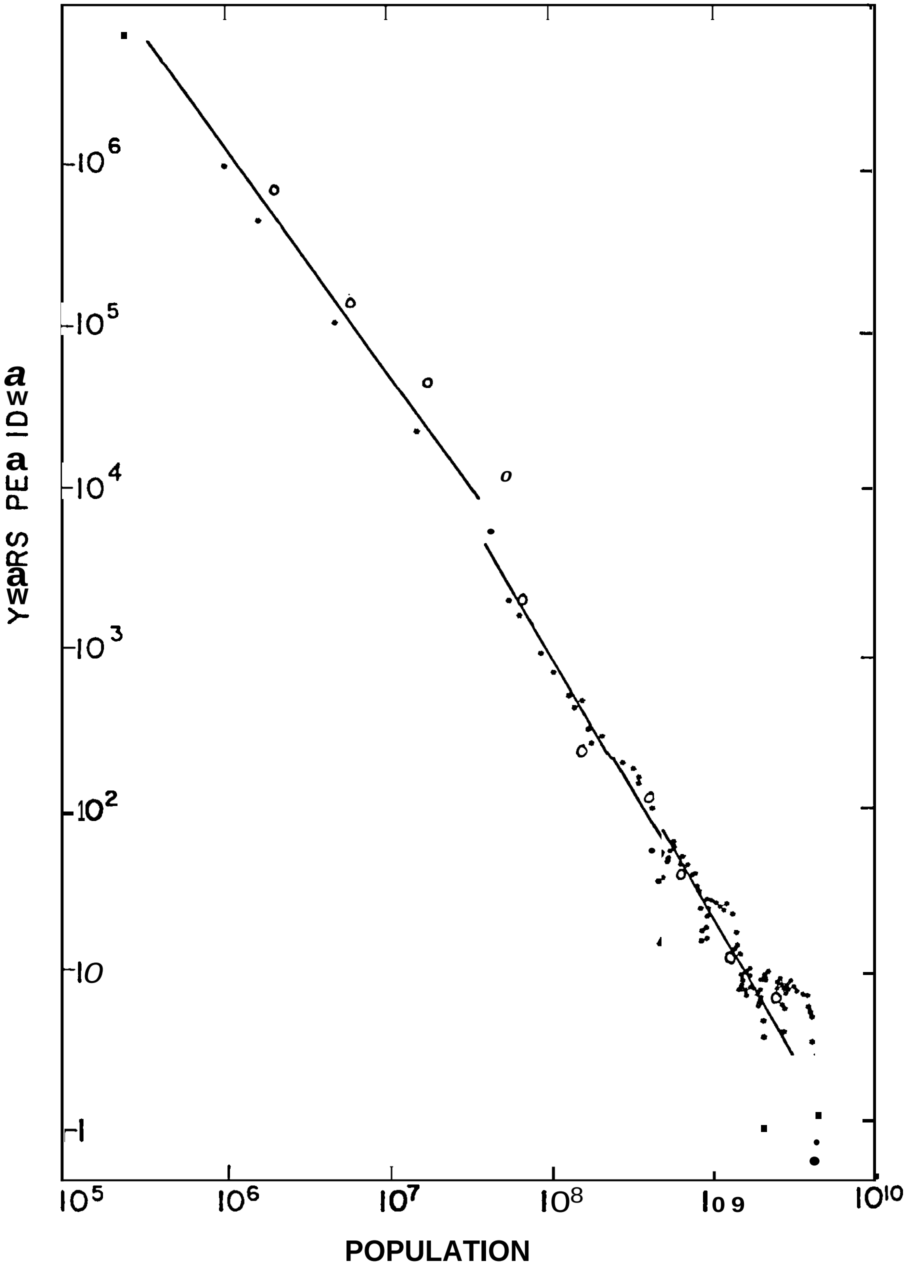

Fig. 1. **Time intervals between ideas plotted against population. Open circles are examples of simple time differences between adjacent ideas. Solid dots are values of dt/dI obtained by finite difference methods, smoothed over five successive ideas. Curve segments are dt/dI from the theory.**

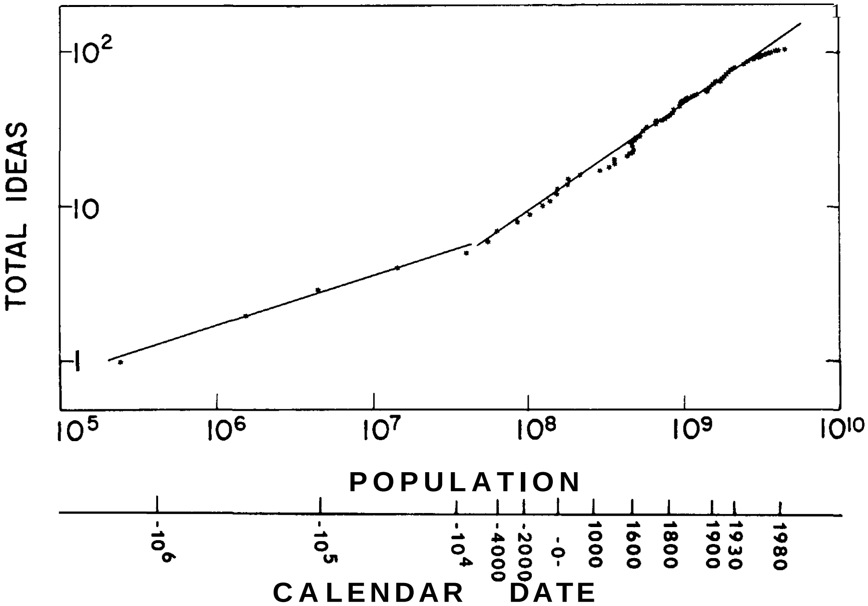

Fig. 2. Accumulated (integrated) number of ideas plotted against population. Note slope change at the fifth idea.

This leads directly to a quantitative reformulation of Henry Adams' hypothesis (cited in the introduction) which must be intuitively obvious to everyone as soon as it is stated in words. The equation is

$$dI/dt = a\alpha IN. \tag{3}$$

Translated, this equation becomes the theorem: "the rate of generation of ideas is proportional jointly to the number of minds to which they can occur, and to the number of ideas already available for those minds to use."

This result can be written in another form:

$$I = \exp\left(a\alpha \int N\,dt\right). \tag{4}$$

This equation makes the slightly different statement that the accumulation of ideas (knowledge) grows exponentially with the number of man years lived.

The differential equation, **Eq. (3)**, for idea growth is the classical model of any random creative collision process such as electrons colliding with atoms to produce new electrons. Analogously, therefore, we might think of idea creation as the result of a "collision" between minds and ideas. The coefficient α is then the probability of the process occurring per person per unit time. The data in Table 1 show that there have been three major changes in this rate throughout history. These data are computed from the values of the

TABLE 1
Population constants

Interval	$1/a\alpha$	$a\alpha$
$-1 \times 10E6$ to -7000	$1.44 \times 10E12$ man-years	$6.93 \times 10E{-13}$/man-year
-7000 to $+1650$	$6.67 \times 10E11$ man-years	$1.50 \times 10E{-12}$ man-year
$+1650$ to $+1930$	$2.76 \times 10E11$ man-years	$3.63 \times 10E{-12}$ man-year

constant a obtained from Fig. 2, and the values of the constant α from the population growth law.

Knowledge of the dependence of I on t enables us to relate the true derivative dt/dI to the experimental data which goes by unit steps. If $\langle a(I)\rangle$ and $\langle k(I)\rangle$ are the local average values of a and k at I fitted over five points, the time interval per idea at I is

$$dt/dI = \frac{\langle a(I)\rangle}{\alpha\langle k(I)\rangle I^s} \tag{5}$$

where $s = 1 + \langle a(I)\rangle$.

If the span of the calculation is over adjacent ideas the irregularity of their spacing makes a very erratic, although not unusable, curve. After trying several values of the number of points in a group, n = 5 was selected. We can now see very definitely that there is a break in the time interval curve at the same point as in the accumulated idea curve. After considering and rejecting the possibility that overlooked ideas or incorrect estimates of population could account for the break, the nature of the discovery at the break came to my attention. It was the invention of writing. The felicitousness of this fact seems hardly accidental. Writing definitely increased the probability that a wider range of minds would interact with the existing ideas. The data indicate that the interaction increased by a factor of two. A priori one might expect that a similar acceleration would have occurred at the invention of printing, but such was not the case, from which we might conclude that dissemination of ideas over space was already largely adequate, and not as important as propagation through time. Although there was no break in the slope of the accumulated idea curve around the 15th century, as plotted against population, there would have been a very sharp break if it had been plotted against time. This shows itself as a second break in the idea rate curve and was caused by the acceleration in population growth at this time rather than by a change in the idea process itself. It is tempting to suggest that the development of printing was a technological advance, and as such had its influence upon technology rather than science directly, and hence in some complex manner led to the mediaeval change in population growth rate which in turn altered the idea growth rate. Ideas per year increased in 1500, but ideas per person did not.

We may now complete the study of the data by plotting the analytical expression for dt/dI upon the experimental points, as shown by the solid lines in Fig. 1. This clearly reveals the segmented character of the data, which we can now see was indicated by the original roughly calculated points also.

Conclusion

What might be the basis for the inference that idea growth in Physics has depended on the totality of minds everywhere, when we all know that most of the recent advances have taken place in the Occident among a few highly educated people? In the matter of universality, it is easy to show that communication has been much more rapid than is often supposed over the whole globe for perhaps a million years. And before one restricts his attention to the unknown numbers of the highly educated he should consider that their numbers are probably proportional to the total, and that the unseen labors of men in the most remote reaches of the globe have provided stimulus, means, and incentive to many if not most of the discoveries made in the main spheres of activity. Finally, the list contains only the first magnitude ideas, but innumerable lesser ones which may have had their origin all around the world went into their conception, and even many of the major ideas themselves have a non-Occidental origin. A more controversial observation from the accumulated idea curve (Fig. 2) is that since 1920 there has been a marked and increasing deficiency of ideas. As of 1980 a full third as many ideas are lacking as are in the entire list! There have often been suggestions that our model of Nature was approaching completeness. But retrospective analysis of the last wave of this feeling, post-Maxwell, shows that at that time a deficiency of 10% at most was in evidence. Earlier, post Aristotle, there was a period when the deficiency reached 20%. The data here seem to show that saturation might indeed now be at hand. And if the limits on particle physics (i.e., on the number of possible neutrinos) which recent analysis (Turner & Schramm, 1979) suggests are validated, the toes of the skeleton of Physics may at last be in sight. On the other hand it is entirely possible that the list is defective in the recent epoch owing to inadequate perspective, although it would seem difficult to overlook about 35 ideas of the stature of the discovery of the neutron, let us say. But with that possibility in mind another revision was undertaken to include all elements of the nuclear and particle problems that seemed to parallel the growth of electromagnetic theory (marked # in the Appendix). Still it was not possible to exorcise the saturation demon. Perhaps whole constellations of solid state and low temperature discoveries of great moment have been omitted, but many other scientists seem to share in the author's belief that these are interesting and extremely useful but not salient matters of Physics. Perhaps on the other hand the word fundamental has at last achieved a full meaning with the emergence of quarks and leptons. Nevertheless, it would be wrong to lay any great stress upon this observation, nor can it be further tested without danger of special pleading. The modicum of objectivity which went into the development of the original list given as Table 1 is now dispersed, and we must await a fresh attack on the problem.

Summary

In summary, the central body of the idea list, running from –5000 to around 1920 deals with a period in which it might be argued that we have a fairly clear idea of which ideas were significant. In this range we find good support for the theorem proposed: that the rate of growth in the number of ideas is proportional jointly to the number of ideas in hand and the number of minds to consider them. Although fluctuations in the growth rate have occurred from time to time, there is no evidence for the theory of Rainoff that it varies periodically.

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APPENDIX A

Roster of ideas, concepts, and discoveries

Event	Date	Population	dt/dI
1 Tools	$-1.5\ 10^{+06}$	$3.2\ 10^{+05}$	$5.6\ 10^{+06}$
2 Abstract language	−300000	$1.6\ 10^{+06}$	$4.7\ 10^{+05}$
3 Counting*	−100000	$4.6\ 10^{+06}$	84000
4 Control of fire	−30000	$1.5\ 10^{+07}$	25000
5 Measurement of space	−9000	$4.1\ 10^{+07}$	9800
6 Writing*	−6000	$5.5\ 10^{+07}$	4500
7 Measurement of time	−5000	$6.3\ 10^{+07}$	1900
8 Metallurgy	−3000	$8.5\ 10^{+07}$	1200
9 Wheel	−2000	$1.0\ 10^{+08}$	750
10 Electricity	−1100	$1.3\ 10^{+08}$	500
11 Magnetism	−800	$1.4\ 10^{+08}$	370
12 Zero	−500	$1.5\ 10^{+08}$	270
13 Concept of "nature"	−400	$1.6\ 10^{+08}$	280
14 Mechanics	−200	$1.7\ 10^{+08}$	320
15 Sound as waves	−100	$1.8\ 10^{+08}$	350
16 Chemical change	400	$2.2\ 10^{+08}$	280
17 Refraction of light	1000	$3.0\ 10^{+08}$	170
18 Observability of nature	1200	$3.4\ 10^{+08}$	79

APPENDIX A (continued)

Event	Date	Population	dt/dI
19 Continuity of space-time	1300	3.7 10^{+08}	58
20 Validity test for theory	1300	3.7 10^{+08}	41
21 Heliocentrism	1543	4.5 10^{+08}	28
22 Dichotomy of charge	1550	4.6 10^{+08}	18
23 Principle of equivalence*	1589	4.8 10^{+08}	11
24 Dipolarity of magnetism	1600	4.8 10^{+08}	6.5
25 Optical instruments	1608	4.7 10^{+08}	5.5
26 Dispersion	1611	4.8 10^{+08}	8.0
27 Planetary laws	1618	4.8 10^{+08}	11
28 Acceleration	1638	5.1 10^{+08}	12
29 Gravitation	1666	5.5 10^{+08}	12
30 The calculus	1667	5.5 10^{+08}	12
31 Finite light speed*	1676	5.6 10^{+08}	12
32 Concept of mass	1687	5.8 10^{+08}	13
33 Action–reaction	1687	5.8 10^{+08}	16
34 Insulators	1730	6.7 10^{+08}	17
35 Duality of electric charge	1733	6.8 10^{+08}	17
36 Fluid mechanics	1755	7.3 10^{+08}	15
37 Electrochemistry	1776	7.9 10^{+08}	13
38 Inverse square laws	1785	8.2 10^{+08}	9.5
39 Generalized coordinates	1788	8.3 10^{+08}	7.1
40 Current electricity	1800	8.8 10^{+08}	5.5
41 Photochemistry	1801	8.8 10^{+08}	4.8
42 Diffraction	1803	8.9 10^{+08}	4.7
43 Atomic chemistry*	1803	8.9 10^{+08}	4.5
44 Spectroscopy	1820	9.6 10^{+08}	4.0
45 Electromagnetism	1820	9.6 10^{+08}	3.6
46 Fourier series*	1822	9.7 10^{+08}	3.8
47 Electrodynamics	1823	9.8 10^{+08}	5.1
48 Ideal engine	1824	9.8 10^{+08}	6.6
49 Electromagnetic induction	1831	1.0 10^{+09}	8.5
50 Energy conservation	1848	1.1 10^{+09}	9.4
51 Kinetic theory*	1857	1.2 10^{+09}	9.8
52 Displacement currents	1864	1.2 10^{+09}	9.0
53 Statistical mechanics	1871	1.3 10^{+09}	7.5
54 Balmer's series	1885	1.4 10^{+09}	5.6
55 Null ether drift	1887	1.4 10^{+09}	4.6
56 Radio waves	1887	1.4 10^{+09}	3.5
57 Photoelectricity	1888	1.4 10^{+09}	2.6
58 X-rays	1894	1.5 10^{+09}	2.2
59 Zeeman effect	1896	1.5 10^{+09}	2.3
60 Radioactivity	1896	1.5 10^{+09}	2.2
61 Electron	1897	1.5 10^{+09}	2.3
62 Photon	1901	1.6 10^{+09}	2.4
63 Mass-energy equivalence	1905	1.6 10^{+09}	2.5
64 Stimulated emission*	1905	1.6 10^{+09}	2.4
65 Geiger counter#	1908	1.7 10^{+09}	2.2
66 Nuclear atom	1911	1.7 10^{+09}	2.0
67 Superconductivity*	1911	1.7 10^{+09}	2.4
68 Cloud chamber#	1912	1.7 10^{+09}	2.8
69 General relativity#	1915	1.8 10^{+09}	2.8
70 Parity#	1922	1.9 10^{+09}	2.5
71 DeBroglie waves	1924	2.0 10^{+09}	2.2
72 Electron spin	1925	2.0 10^{+09}	1.5

APPENDIX A (continued)

Event	Date	Population	dt/dI
73 Exclusion principle	1925	2.0 10^{+09}	1.1
74 Artificial radioactivity	1925	2.0 10^{+09}	1.0
75 Beta-ray dilemma	1927	2.0 10^{+09}	1.1
76 Electron diffraction#	1927	2.0 10^{+09}	1.3
77 Cyclotron#	1930	2.1 10^{+09}	1.5
78 Superfluidity*	1932	2.1 10^{+09}	1.8
79 Neutron	1932	2.1 10^{+09}	2.2
80 Positron	1932	2.1 10^{+09}	2.7
81 Mu meson	1936	2.2 10^{+09}	3.1
82 Pi meson	1947	2.5 10^{+09}	2.8
83 Controlled semiconduction*	1948	2.6 10^{+09}	2.3
84 Quantum electrodynamics*	1949	2.6 10^{+09}	1.9
85 Computers*	1949	2.6 10^{+09}	1.7
86 Strong focussing#	1952	2.7 10^{+09}	1.2
87 Neutrino detection*	1953	2.7 10^{+09}	1.3
88 Bubble chamber#	1953	2.7 10^{+09}	1.5
89 Antiproton	1955	2.8 10^{+09}	1.8
90 Parity violation	1956	2.9 10^{+09}	1.7
91 BCS theory	1957	2.9 10^{+09}	1.9
92 Strangeness	1958	3.0 10^{+09}	2.7
93 Big bang#	1963	3.2 10^{+09}	2.8
94 Symmetry theory	1964	3.2 10^{+09}	2.6
95 Laser*	1968	3.5 10^{+09}	2.4
96 Josephson effect*	1971	3.7 10^{+09}	2.1
97 J/psi particle	1974	3.9 10^{+09}	1.3
98 Quark structure of nucleons*	1975	4.0 10^{+09}	.93
99 Color force#	1976	4.1 10^{+09}	.65
100 Color multiplicity#	1976	4.1 10^{+09}	.76
101 Quantum chromodynamics*	1976	4.1 10^{+09}	.91
102 Flavor#	1977	4.1 10^{+09}	1.0
103 Grand unified theory*	1979	4.3 10^{+09}	1.2
104 Gluon*	1979	4.3 10^{+09}	1.6
105 W particles#	1982	4.6 10^{+09}	1.7

Note. The symbol * denotes a concept that was added on a second searching scrutiny to achieve greater completeness. The symbol # prior to about 1963 refers to an idea added in a conscious effort to overcome the apparent deficiency in the number of ideas since 1920. After 1963, it refers to an attempt to add ideas on nuclear and particle physics which parallel the growth of electromagnetic theory.